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Coordinating strategy of supply chain contract based on price discount and quantity buyback

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Abstract: The coordinating decision model of supply chain contract is developed with the contracts of price discount and quantity buyback, irrespective of shortage cost. It obtains the optimal ordering strategy of retailer and quantity buyback policy of supplier. Finally, numerical example is given to illustrate the feasibility of this model, and a conclusion is obtained that corresponds to practice.

Keywords: Price Discount, Quantity Buyback, Supply Chain Contract

1. Introduction

Pasternack (1985) first brought up the concept of supply chain contract. It refers to a kind of articles through the provision of appropriate information and incentives. It could ensure coordination of buyers and sellers, and optimize the performance of the relevant provisions of the sales channels. It is a form of expression of economic contract theory in supply chain. Then, some studies on supply chain contract have a considerable progress in many aspects. It falls into four major types: wholesale price contract, buyback contract, revenue sharing contract and quantity flexibility contract. Among them, the wholesale price contract and buyback contract were studied early and they were the most common type of contract, while revenue sharing contract and quantity flexibility contract studied the core of the supply chain: member revenue and product quantity.

In Supply Chain Management (SCM), suppliers and sellers form a supply chain alliance through the contract and they constitute the Stackelberg Game. The supplier is the host, the seller is secondary. The suppliers maximize the benefit through the reasonable contract, and the sellers maximize their income through hard-working after accepting the contract. Therefore the ways they obtaining the profit are inconsistent. In supply chain, decentralized decision-making model is more common than the mode of centralized decision-making. The contact between the supply chain nodes is the link to build the supply chain. The uncertainty of market leads to a lack of coordination. Starting from the uncertain market demand, through the research of supply chain contract, it can achieve the coordination of supply chain. It is an important part of the supply chain study. In SCM, the basic structure is the secondary single-cycle supply chain behavior composed of suppliers and sellers. The study on the contract and coordination of this basic structure constitutes the basic elements of SCM research. Price discounts and quantity buyback is an important tool to research the contract and the coordination.

Pasternack (1985) proposed the Quantity Buyback which is used to coordinate the behaviors of suppliers and sellers. Cachon (1999) has studied that under the sole wholesale price the supplier use quantity buyback in order to enable the order amount achieve the demand quantity which keep the supply chain coordinated. In the decentralized decision-making model of supply chain, it is obvious for the seller that the supplier wants to maximize the benefit through the contract. It is clear that the sellers not only have to strive for the advantageous provision in the contract stipulation, but also work hard if they want to make the income maximization. Thus quantity buyback contract relates to the inspiration of sellers' hard working. Larivece (1999) studied the relationship of quantity buyback strategy and incentive. Krishnan (2004), Zhang Juliang (2004) and Chenjian (2006) also discussed the incentive problems in the supply chain.

On the basis of those literatures above, this paper revises some assumptions of studies, establishes coordinating strategy of supply chain contract for distributors which is under the circumstance that the suppliers fix the price discounts and the quantity buyback, thus further improve coordinating model of supply chain. This makes the model more practical, and lets suppliers and the sellers know how to calculate the maximization of benefits in the two circumstances.

2. Symbols and Assumptions Used in the Model

Supply chain contract (generally speaking) is a two-stage supply chain which is composed of one supplier (manufacturer) and one distributor (retailer) shown in Figure 1.

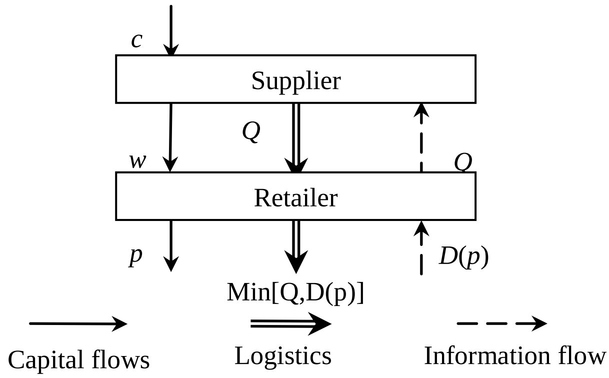

Figure 1: Typical supply chain model

Retailers face a market in which the demand is random. Products are seasonal and order a long period. According to the theory study of LF Game (Leader-Follower Game) on the interaction between suppliers and retailers, suppliers are the leaders and retailers follow them. Suppliers set contract parameters, whereby retailers determine optimal order quantity of products. At the same time it shows that the market is open. Such information as the product prices of related market, demand distribution and inventory cost is symmetric.

Therefore, as a leader, a vendor can get all the necessary information, infer the order quantity of retailer, and thus make the best decision. Suppliers and retailers are risk-neutral or fully rational. They are based on the principle of expected maximum profit.

In order to study easily, the symbols of the model used in the paper are shown in Table 1.

Table 1: Symbols of the mode

Symbol	Meaning	Symbol	Meaning
Q	Each ordering of the retailer	v	Each price of retailer deals stock, $v < c$
D	Market demand	r	Price of supplier buyback from retailer, $r > v$
μ	Mean of the market demand	β	Ratio of return and order, $0 < \beta < 1$
c	Each production product cost	$S(Q)$	Expectant sales of retailer
c_t	Each production sales cost	$I(Q)$	Expectant stocks of retailer
w	Trade price of supplier to retailer	Π_R	Expectant returns of retailer

p	Each production retail cost	Π_S	Expectant returns of supplier
c_e	Each stock cost	Π_S	Expectant returns of supplier
q	Production quantity supplier products	Q^*	Balanced product quantity
$F(x)$	Demand distribution function, $x \in [0, +\infty)$	$F(Q^*)$	Balanced product quantity function
$f(x)$	Probability density function, $x \in [0, +\infty)$	Q_R^*	Optimal order quantity of retailer

The paper studies on the two-stage supply chain contract system which is composed of one supplier and one retailer and only focuses on the operation of a single product cycle. The assumptions of the model are given as follows: (1) the market demands of products $D \geq 0$, the market price of products p the same. (2) demand distribution function $F(x)$ is continuous and differentiable; (3) Supply chain implement a distributed decision-making; (4) time interval is infinite; (5) do not consider the losses of out of stock.

3. Construction of the Model

3.1. the Basic Model of Supply Chain Contract

Based on these assumptions above, there are establishments of the following relationship:

$$u = \int_0^{+\infty} x f(x) dx \qquad (1)$$

$$\begin{aligned} S(Q) &= \int_0^{+\infty} (Q \wedge x) f(x) dx \\ &= \int_0^{+\infty} \int_0^{Q \wedge x} dy f(x) dx \\ &= \int_0^{Q} \int_y^{+\infty} f(x) dx dy = \int_0^{Q} F(x) dx \end{aligned} \qquad (2)$$

Among the formula， $\wedge$ means that it take the small one in two numbers.

The inventory function expected by retailers:

$$I(Q) = E(Q - x) = Q - S(Q) \qquad (3)$$

Profit function expected by retailers:

$$\Pi_R = pS(Q) + vI(Q) - c_e I(Q) - wQ \qquad (4)$$

Profit function expected by suppliers:

$$\Pi_S = (w - c)Q \qquad (5)$$

The expectations of the whole supply chain profit function as:

$$I(Q) = E(Q - x) = Q - S(Q) \qquad (6)$$

According to (4), (5) and (6) get:

$$\Pi_T = (p + c_e - v)S(Q) - (c + c_e - v)Q \qquad (7)$$

According to the Leibniz rule, we can see Π_T is a concave function, and Π_T gets partial differential coefficient of Q:

$$\begin{aligned} \frac{\partial \Pi_T}{\partial Q} &= (p + c_e - v)S'(Q^*) + v - c_e - c \\ &= (p + c_e - v)F(Q^*) + v - c_e - c = 0 \end{aligned} \qquad (8)$$

The equilibrium order functions of supply chain contract:

$$F(Q^*) = \frac{p - c}{p + c_e - v} \qquad (9)$$

The above formula has got inverse function. That is the balanced order quantity.

$$Q^* = F^{-1}\left(\frac{p - c}{p + c_e - v}\right) \qquad (10)$$

The optimal order quantity of retailers:

$$Q_R^* = \arg\max \Pi_R \qquad (11)$$

From function (11), we can find that researches of collaborating supply chain contract focus on two major points: First, the retailer order quantity to the optimal efficiency of the supply chain; second, how suppliers and retailers allocate the profits of the supply chain.

3.2. Contract Model with Price Discount

When there is price discount, then use the wholesale price contract. The retailer's profit function is:

$$\begin{aligned} \Pi_R &= pS(Q) + vI(Q) - c_e I(Q) - wQ \\ &= (p + c_e - v)S(Q) - (w + c_e - v)Q \end{aligned} \qquad (12)$$

Similarly, according to Leibniz rule, we can see Π_R is a concave function. Π_T gets partial derivative of Q ， and make the equation equal to zero, we obtain the optimal order quantity for the retailer:

$$Q_R^* = F^{-1}\left(\frac{p - w}{p + c_e - v}\right) \qquad (13)$$

In order to achieve coordination, it must be $Q_R^* = Q^*$, $w = c$. Now the supplier will not gain profit. It is certainly wrong in the common sense. Therefore, a simple wholesale price contract cannot coordinate the supply chain.

3.3. the Contract Model of Taking into Account the Price Discount and Quantity Buyback

Spengler (1950) first discovered that a simple wholesale price contract cannot achieve coordinating supply chain mainly due to double marginal benefit of suppliers and retailers. They only consider the marginal benefits unilaterally, without considering the overall marginal benefits. Even so, the price discount contact has been widely used in practice because it is simple and less costly. As for the double marginalization of suppliers and retailers, Chao and others have made a more detailed analysis on it.

There are many studies on quantity buyback contract too. Padmanabhan (1997), Tagaras (1992), Anupindi (2001) et al have studied on the motivation of quantity buyback contract. Padmanabhan (1997) et al studied the following situation: in order to control the excessive competition among retailers, suppliers adopt the quantity buyback strategy. Thus they found that suppliers gain more. But in the case of fixed demand, quantity buyback strategy will result in non-rational order quantity. To solve the problems above, the price discount and quantity buyback contract model will lead to the maximization of marginal benefit of whole supply chain. When there is a quantity buyback strategy, that is, suppliers repurchase back the remaining products with supply a reasonable commercial price $r(r > v)$, so as to stimulate retailers to increase the order quantity and expand product sales. Now the retailer's profit function is:

$$\begin{aligned}\Pi_R &= pS(Q) + rI(Q) - wQ - c_t S(Q) \\ &= pS(Q) + r\beta(Q - S(Q) - wQ - c_t S(Q) \\ &= (p - r\beta - c_t)S(Q) - (w - r\beta)Q\end{aligned} \qquad (14)$$

The retailer's profit function is:

$$\begin{aligned}\Pi_S &= (w-c)Q - r\beta(Q - S(Q)) \\ &= (w - c - r\beta)Q + r\beta S(Q)\end{aligned} \qquad (15)$$

The expectations of the whole supply chain profit function:

$$\begin{aligned}\Pi_T &= \Pi_R + \Pi_S \\ &= \begin{cases} pQ - (c + c_t)Q & Q \le F(x) \\ pQ - (c + c_t)Q - p(Q - F(x)) & Q > F(x) \end{cases}\end{aligned} \qquad (16)$$

Under the above assumptions, in order to achieve the coordination state of the supply chain, suppliers choose r , w , β value, and make the expected value of Π_R maximum under the conditions of coordinating supply chain . Sellers choose Q and make the expected values of Π_S maximum. From (16), the overall revenue of supply chain is decided by the order quantity of sellers. The parameter $\{r,w,\beta\}$ is the parameter for optimal decision.

4. Model Solving and Coordination Analysis

4.1. Model solving

From (14) and (15), we know, the expected profit of suppliers and vendors is composed of two parts: the first part is the sales revenue when the order quantity is Q; the second part is production, sales and ordering costs. From (16), we can see that the revenue of the whole supply chain only depends on the ordering quantity, nothing to do with the buyback strategy.

Similarly, according to Leibniz rule, we can see Π_R is a concave function. Π_T gets partial derivative of Q , and make the equation equal to zero:

$$\begin{aligned}\frac{d\Pi_R(Q)}{DQ} &= (p-r\beta-c_t)S'(Q^*)-(w-r\beta) \\ &= (p-r\beta-c_t)F(Q^*)-(w-r\beta)=0\end{aligned} \tag{17}$$

The retailer order function:

$$F(Q_R^*)=\frac{w-r\beta}{p-c_t-r\beta} \tag{18}$$

Retailer's optimal order quantity is:

$$Q_R^*=F^{-1}\left(\frac{w-r\beta}{p-c_t-r\beta}\right) \tag{19}$$

Similarly, according to Leibniz rule, we can see Π_T is a concave function, Π_T gets partial derivative of Q, and make the equation equal to zero:

$$\begin{aligned}\frac{d\Pi_T(Q)}{DQ} &= (p-r\beta-c_t)S'(Q^*)-(w-r\beta)+ \\ &(w-c-r\beta)+r\beta S'(Q^*)=0\end{aligned} \tag{20}$$

The supply chain contract equilibrium order function:

$$F(Q^*)=\frac{c}{p-c_t} \tag{21}$$

Balanced order quantity of supply chain is:

$$Q^* = F^{-1}(\frac{c}{p - c_t}) \qquad (22)$$

In order to achieve coordination, it must :

$$w = r\beta + \frac{c(p - c_t - r\beta)}{p - c_t} \qquad (23)$$

Put (23) into equation (14), after finishing, it can get retailers profit:

$$\Pi_R = \frac{p - c_t - r\beta}{p - c_t}\Pi_T \qquad (24)$$

The profit of suppliers is:

$$\Pi_S = \Pi_T - \Pi_R = \frac{r\beta}{p - c_t}\Pi_T = \eta\Pi_T \qquad (25)$$

In the equation, $\eta = \dfrac{r\beta}{p - c_t}$.

Because of $0 < \dfrac{r}{p - c_t} < 1$, $0 < \eta < 1$, so the quantity buyback strategy can achieve coordinating supply chain. According to equation (25), through selecting the repurchase price of the size and the proportion of quantity buyback, vendors determine their own possession of the entire supply chain profit share, and according to the type (23) to determine the optimal price w .

In order to coordinate the supply chain and also enables suppliers maximize their profit, the parameter of quantity buyback contract fixed by vendors should be optimal solution of the following planning problems.

$$\begin{aligned} &\max_{r,w,\beta} E(\Pi_S) = wQ - c_t Q - r\int_{(1-\beta)Q}^{Q} F(x)dx \\ &s.t. \quad c_t = p - w - (p - r)F(Q) \\ &\qquad\qquad - (1-\beta)rF((1-\beta)Q) \\ &\qquad\qquad 0 \le \frac{r}{w} \le 1 \\ &\qquad\qquad 0 \le \beta \le 1 \end{aligned} \qquad (26)$$

The equation (17) is the condition of retailers' profit maximization, and the vendor must consider their own maximum profit. The equation (26) is the whole profit of the supply chain.

According to the planning problems, get that when $r = w$, $\beta = 1$, the suppliers gain the maximum profit. And from the equation (14) and (21), get:

$$w = p - \frac{c}{1-F(Q^*)} = \frac{c_t}{1-F(Q^*)} \quad (27)$$

The equation (27) shows that in order to coordinate the supply chain and also maximize the overall revenue, suppliers should allow the retailers return all order products that couldn't be sold with the wholesale price.

The formula (27) becomes deformed:

$$p = w + \frac{c}{1-F(Q^*)} \quad (28)$$

From equation (28), it can be seen that the market price p of products is divided into two parts: the first part is w, the second part $\frac{c}{1-F(Q^*)}$ is received by the retailers.

Thus, the optimal decision condition which maximizes the overall profit of supply chain contract is: $r = w$ and $\beta = 1$.

4.2. Examples

The following example shows the feasibility of the model.

Set a retailer to develop a product order policy for a supplier, as the following data:

p =450, w =400, c =150, c_e =30, c_t =15， $S(Q)$ =2000, Q =2600.

When $r = w$ and $\beta = 1$, after calculating available: Π_R =7000, Π_S =740000, at this point, the total profit values of supply chain are: Π_T =810000.

Adjust r and β, we can get other results, concrete results are shown in table 2.

Table 2: Effect of whole supply chain profit according to the change of γ and β

r	β	Π_R	Π_S	Π_T	Conclusion
350	0.95	29500	699500	729000	<810000
380	0.85	18700	688700	707400	<810000
410	0.80	26800	696800	723600	<810000

From the analysis of available data in Table 2,we can get a such result: As the supplier to retailer's number returns the value of the contract increase r , Even more than the wholesale price of repurchase, In the not too suppliers profit values, The retailer's return values appear larger trends, But the vendor number returns policy should be raised with the repurchase price β should be reduced, Or you may make vendor profit decline, But no matter what combination of strategy, Still in $r = w$ and $\beta = 1$, Supply chain's total profit value is the maximum, At this point, the supply chain to achieve better coordination. Therefore the model and conclusions of the paper correspond with the practice.

5. Conclusion

In the supply chain contract, price discount contract has been extensively used in practice because of its simple and low cost of implementation. But it doesn't overall consider the marginal benefit of the entire supply chain; in the case of fixed demand, quantity buyback contract will lead to the retailer's order irrational. Therefore, without considering the loss out of the premise, the paper established a supply chain management model which is a single-cycle behavior and contains only one supplier and one retailer. It contains two contractual relationships of price discounts and quantity buyback. This kind of model is more close to reality. Model analysis shows that the contract can achieve supply chain coordination. Meanwhile, it verifies the feasibility of the model and the conclusions match to the reality by examples calculation.

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